Int. J. Heat Mass Transfer. Vol. 35, No. 2, pp. 351–359, 1992
Printed in Great Britain

0017–9310/92 $5.00+0.00

Experimentally determined turbulent Prandtl numbers in liquid sodium at low Reynolds numbers

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(*Received* 6 *November* 1989 *and in final form* 5 *April* 1991)

Abstract—Turbulent Prandtl numbers have been deduced from measurements of mean velocity and mean temperature distributions and their half value radii for a liquid sodium jet discharging into essentially still fluid. Results obtained indicate a velocity dependence and are similar to those reported in the literature for pipe flows. The velocity dependence, which is assumed to be due to transition to an inertial-conductive flow regime where turbulent diffusion of heat inherent in the turbulent Prandtl number no longer exists, becomes pronounced at low velocities. Combination of the new jet flow and existing pipe flow data for liquid sodium leads to a simple functional relationship between turbulent Prandtl number and eddy diffusivity of heat which may be readily solved for ε_H once ε_M is known. The combined results show that, for liquid sodium, significant increases in turbulent Prandtl number occur when flow conditions lead to $\varepsilon_H/\alpha < 2.5$, that is, when $\varepsilon_M/\nu < 2/Pr$.

INTRODUCTION

INVESTIGATIONS into heat transfer in a Liquid Metal Cooled Fast Breeder Reactor (LMFBR) are often carried out using water as the working fluid, as this allows a much simpler design of test facilities. Consequently, results of water experiments serve as a basis for the verification of computer codes developed for the determination of decay heat removal after shutdown of an LMFBR [1].

These codes make use of turbulence models which are based on the ratio of the eddy diffusivity of momentum, ε_M, to the eddy diffusivity of heat, ε_H, that is, the turbulent Prandtl number $Pr_t = \varepsilon_M/\varepsilon_H$. For air and water, Pr_t is assumed to be 0.9 independent of velocity, whereas fluids with a low molecular Prandtl number, such as liquid metals, show a velocity dependence [2].

Numerous reviews are available in the literature outlining the problem of turbulent heat transfer prediction from a knowledge of the velocity field and the turbulent Prandtl number. The most relevant of these for low Prandtl number flows are by Jischa and Rieke [3] and Jischa [4]. These authors also attempt to obtain a turbulent Prandtl number expression by use of the transport equation for the turbulent heat flux. By assuming that production and dissipation dominate the turbulent heat flux balance, the following result is obtained:

$$Pr_t = K_1 + \frac{K_2}{Pr\,Re^m}. \quad (1)$$

Application of this result to fully developed, heated wall pipe flow data yielded values of $K_1 = 0.9$, $K_2 = 182.4$ and $m = 0.888$. If turbulence Reynolds number is used instead of Re in equation (1), m would be unity and K_2 would change, but the basic Reynolds number dependence still exists.

Sheriff and O'Kane [2] measured turbulent Prandtl numbers at the core of fully developed pipe flow by use of a point source of hot sodium located at the pipe centre line. Such an arrangement leads to non-zero convective terms for the turbulent heat flux and hence does not completely satisfy the assumption that production of turbulent heat flux is balanced by its dissipation. The eddy diffusivities, and hence turbulent Prandtl numbers, obtained were similar to those reported by Fuchs [5], who used wall heating. The results of Sheriff and O'Kane [2] indicate that when $\varepsilon_H/\alpha < 0.25$, Pr_t increases rapidly towards infinite values as velocity is decreased, whereas those of Fuchs [5] approach a limit of 2.6.

Convective effects are even more pronounced in free jets. Suitable data for jet flows at low Prandtl numbers, particularly liquid sodium, do not appear to exist. The purpose of this paper is to present such data for comparison with the pipe flow data in order to test for Reynolds number dependence.

NOMENCLATURE

A_1, A_3 constants
A_2, A_4 constants [m^{-2}]
d jet exit diameter [m]
k turbulent kinetic energy [$m^2 s^{-2}$]
P pressure [Pa]
Pr molecular Prandtl number
Pr_t turbulent Prandtl number
r radial coordinate [m]
$r_{1/2,T}$ half value radius for radial temperature distribution [m]
$r_{1/2,U}$ half value radius for radial velocity distribution [m]
Re Reynolds number based on bulk velocity and jet exit diameter
Re_0 Reynolds number based on U_0 and jet exit diameter
T mean temperature above T_∞ [°C]
T_∞ free stream temperature [°C]
u streamwise velocity fluctuation [$m s^{-1}$]
U mean streamwise velocity above U_∞ [$m s^{-1}$]
U_∞ free stream velocity [$m s^{-1}$]
v radial velocity fluctuation [$m s^{-1}$]
V mean radial velocity [$m s^{-1}$]
x streamwise coordinate
$x_{01}, x_{02}, x_{03}, x_{04}$ effective origins [m].

Greek symbols
α molecular thermal diffusivity [$m s^{-2}$]
δ temperature fluctuation [K]
ε_M eddy diffusivity of momentum [$m s^{-2}$]
ε_H eddy diffusivity of heat [$m s^{-2}$]
ν kinematic viscosity [$m s^{-2}$].

Superscripts
$\bar{}$ overbar denotes time averaging
$'$ r.m.s. value.

Subscripts
E evaluated at exit condition of centre bore producing jet flow
k based on turbulent kinetic energy
u' based on u'
0 jet centre line value
∞ free stream condition or flow outside of jet.

BASIC EQUATIONS

Equations of motion

The streamwise Reynolds equation for steady, axisymmetric jets of constant density, free of swirl, discharging into fluid moving with a constant velocity of U_∞, negligible viscous effects and negligible axial relative to radial gradients of turbulent quantities, is given by equation (2) [6], where U is taken relative to U_∞:

$$(U+U_\infty)\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial x}-\frac{1}{r}\frac{\partial(r\overline{uv})}{\partial r}. \quad (2)$$

Introduction of the gradient diffusion assumption for eddy diffusivity of momentum

$$\varepsilon_M = -\frac{\overline{uv}}{\partial U/\partial r} \quad (3)$$

leads to the x momentum equation

$$(U+U_\infty)\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left[r\varepsilon_M\frac{\partial U}{\partial r}\right] \quad (4)$$

where the mean radial velocity V, equation (5b), can be obtained from the continuity equation, equation (5a), by integration:

$$\frac{\partial U}{\partial x}+\frac{1}{r}\frac{\partial (rV)}{\partial r} = 0 \quad (5a)$$

$$V = -\frac{1}{r}\int_0^r r\frac{\partial U}{\partial x}\,dr. \quad (5b)$$

Experimental results indicate that the Gaussian curve, equation (6), gives a good fit:

$$\frac{U}{U_0} = \exp\left[-(\ln 2)\left(\frac{r}{r_{1/2,U}}\right)^2\right] \quad (6)$$

where U_0 is the centre line velocity above U_∞ and $r_{1/2,U}$ is given by

$$\frac{2r_{1/2,U}}{d} = A_2\left(\frac{x}{d}+\frac{x_{02}}{d}\right). \quad (7)$$

If the decay of U_0 follows

$$\frac{U_0}{U_E} = \frac{A_1}{\left(\frac{x}{d}+\frac{x_{01}}{d}\right)} \quad (8)$$

where U_E is the velocity above U_∞ at the jet exit, then integration of equation (4) and the associated radial momentum equation for P_∞ = constant [6], and use of equations (5)–(8), leads to the rather simple result at the jet centre line that

$$\frac{\varepsilon_{M0}}{U_E d} = \frac{A_1A_2^2}{16(\ln 2)}\frac{\left[1+\frac{U_\infty}{U_E A_1}\left(\frac{x}{d}+\frac{x_{01}}{d}\right)\right]\left[\frac{x}{d}+\frac{x_{02}}{d}\right]^2}{\left[\frac{x}{d}+\frac{x_{01}}{d}\right]^2}. \quad (9)$$

Thermal energy equation

Neglecting dissipation, internal heat generation and radiation heat transfer, the thermal energy equation for the above case becomes, in the Reynolds form for subsonic flow

$$(U+U_\infty)\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial r}=-\frac{\partial \overline{u\delta}}{\partial x}-\frac{1}{r}\frac{\partial(r\overline{v\delta})}{\partial r}+\alpha\left[\frac{\partial^2 T}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\,\partial T}{\partial r}\right)\right] \quad (10)$$

where T is the jet temperature above that of the outer flow, which is at T_∞. Introducing the gradient diffusion assumption for eddy diffusivity of heat

$$\varepsilon_{\rm H}=-\frac{\overline{v\delta}}{\partial T/\partial r} \quad (11)$$

and neglecting axial gradients relative to radial ones on the right-hand side of equation (10) leads to

$$(U+U_\infty)\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left[r(\varepsilon_{\rm H}+\alpha)\frac{\partial T}{\partial r}\right]. \quad (12)$$

It should be noted that the molecular diffusion term has been retained as this is significant for low Prandtl number fluids.

Assuming similar functional forms for temperature variations as for U yields

$$\frac{T}{T_0}=\exp\left[-(\ln 2)\left(\frac{r}{r_{1/2,T}}\right)^2\right] \quad (13)$$

where

$$\frac{2r_{1/2,T}}{d}=A_4\left(\frac{x}{d}+\frac{x_{04}}{d}\right) \quad (14)$$

$$\frac{T_0}{T_{\rm E}}=\frac{A_3}{\left(\frac{x}{d}+\frac{x_{03}}{d}\right)}. \quad (15)$$

Integration of equation (12) and use of equations (13)–(15) again leads to a simple result at the jet centre line

$$\frac{\varepsilon_{\rm M0}+\alpha}{U_{\rm E}d}=\frac{A_1A_4^2}{16(\ln 2)}\frac{\left[1+\frac{U_\infty}{U_{\rm E}A_1}\left(\frac{x}{d}+\frac{x_{01}}{d}\right)\right]\left[\frac{x}{d}+\frac{x_{04}}{d}\right]^2}{\left[\frac{x}{d}+\frac{x_{01}}{d}\right]\left[\frac{x}{d}+\frac{x_{03}}{d}\right]}. \quad (16)$$

Combination of equations (9) and (16) yields the turbulent Prandtl number, $Pr_{\rm t}=\varepsilon_{\rm M0}/\varepsilon_{\rm H0}$.

EXPERIMENTAL EQUIPMENT AND INSTRUMENTATION

Figure 1 gives a schematic of the liquid sodium test facility. The containment pipe of 0.11 m internal diameter provided the ambient flow into which the heated jet was injected. The latter was the central hole of a multibore jet block consisting of 0.0072 m diameter holes placed on a 0.0082 m triangular pitch. Flow was vertically upward and in order to avoid recirculation of the entrained flow, the ambient fluid was given an upward velocity of 0.05 m s^{-1} through the jet block. Ambient fluid was, therefore, at very low turbulence level with minimal natural convective effects. In order to permit measurements at various downstream distances, the jet block could be moved relative to the fixed radial traversing stations.

A newly developed miniature, temperature compensated permanent magnet flow probe of 2.5 mm overall diameter was used to measure temperature and velocity [7]. Temperature sensors in this probe were chromel/alumel thermocouples.

The temperature of the free stream flow was kept at 300°C whereas the temperature of the central jet was 329.5°C at the exit of the jet block. Fluid properties were evaluated at the intermediate temperature of 307°C, thus giving $\nu=3.86\times10^{-7}$ m^2 s^{-1}, $\alpha=6.66\times10^{-5}$ m^2 s^{-1} and a molecular Prandtl number of 0.0058.

EXPERIMENTAL RESULTS

The Gaussian curve fits radial velocity defect (that is, corrected for U_∞) and temperature profiles well for $x/d \geqslant 4$. From these, the jet half value radii $r_{1/2,U}$ and $r_{1/2,T}$—the radii at which U and T are half those at the centre line—were obtained. In order to test for a velocity or Reynolds number effect, measurements were performed at two jet exit velocities.

Centre line velocities (Fig. 2) are seen to follow closely the inverse decay law of equation (8). Comparisons with other data are included. The data of Corrsin [8] and Wilson and Danckwerts [9] were obtained in a heated jet of air whereas those of Kiser [10] were obtained in water. The general correlation presented by Chen and Rodi [11] is also included. Any lateral shift of the curves due to differing effective origins can be ignored as such a shift depends on nozzle exit details and exit conditions. The slopes of trend lines through various data are seen to be in general agreement, although the data of Corrsin [8] and Kiser [10] also deviate somewhat from the linear relationship. Although the half value radii of velocity show some scatter (Fig. 3), the trends are in good agreement with the results of others and the form of equation (7).

Decay of centre line temperatures (Fig. 4) is compared with data by Corrsin [8], Wilson and Danckwerts [9] and the correlation recommended by Chen and Rodi [11]. In the case of the data of Wilson and Danckwerts [9], the anomalous low temperature result has been ignored. Corresponding temperature half value radii and the best fit of equation (14) are shown in Fig. 5, together with the values of Corrsin [8], Wilson and Danckwerts [9] and Chen and Rodi

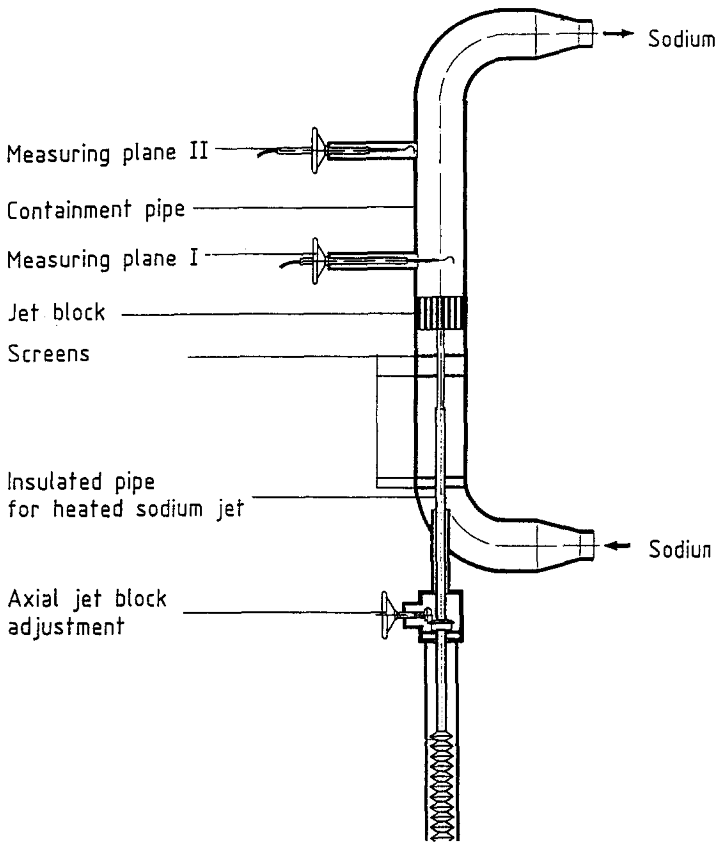

FIG. 1. Schematic of sodium test section.

[11]. Except for the data of Wilson and Danckwerts [9], good agreement is indicated.

Application of equation (16) to the results of Figs. 2–5 leads to the estimate of the turbulent eddy diffusivity of heat with downstream distance as given in Fig. 6. The range of ε_{H0}/α covered in the present experiments lies somewhat higher than that by previous investigators in pipe flows [2, 5], but it is seen to be in the range where the turbulent eddy diffusivity of heat and the molecular diffusivity are of nearly equal importance.

Equations (9) and (16) may be used with the results of Figs. 2–5 to give an estimate of turbulent Prandtl number (Fig. 7). These results indicate only a weak dependence of Pr_t on local velocity and hence x/d but a distinct increase is observed with decreasing velocity

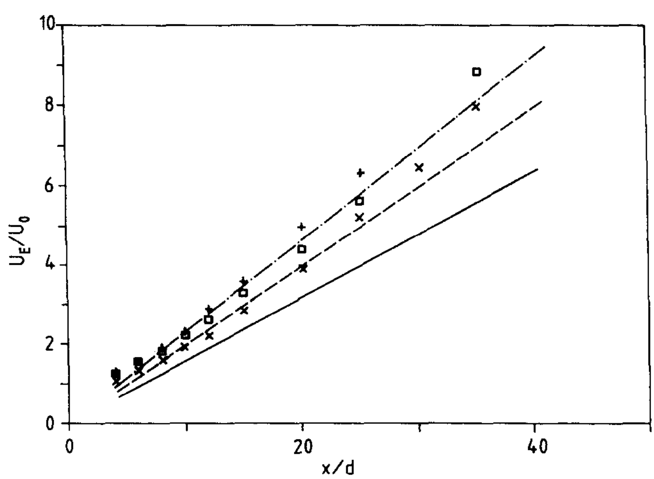

FIG. 2. Centre line velocity decay. (+) $U_E = 0.75$ m s^{-1}; (□) $U_E = 1.25$ m s^{-1}; (×) Kiser [10]. (–·–) Equation (8), $A_1 = 4.3$, $x_{01}/d = 0$. (– –) Wilson and Danckwerts [9]. (——) Chen and Rodi [11].

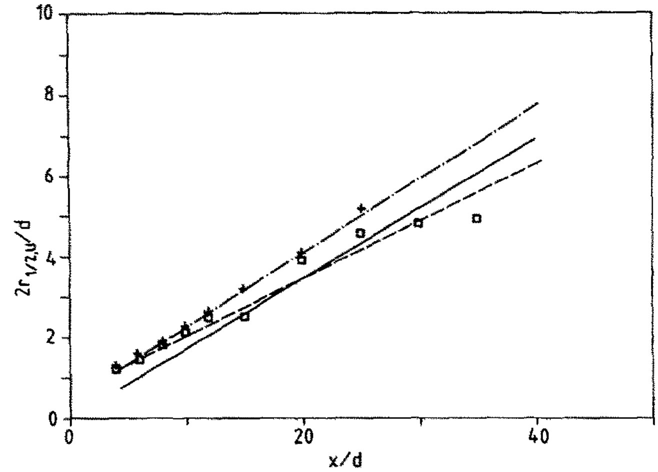

FIG. 3. Half value radii of velocity profiles. (+) $U_E = 0.75$ m s^{-1}; (□) $U_E = 1.25$ m s^{-1}. (-·-) Equation (7), $A_2 = 0.183$, $x_{02}/d = 2.4$. (--) Equation (7), $A_2 = 0.143$, $x_{02}/d = 4.4$. (——) Chen and Rodi [11].

for the low velocity jet. The rapid rise in Pr_t with decreasing velocity and hence decreasing Reynolds number reported by Sheriff and O'Kane [2] and predicted by Jischa [4] is not observed, probably because the present measurements could not be extended to sufficiently low values of ε_{H0}/α.

Since turbulent heat transfer is a diffusive process governed by the action of the turbulence, it seems more appropriate to use a turbulent Reynolds number, $Re_{u'}$, as already given by Jischa [4], so that comparison of different types of turbulent flows becomes possible. Using the jet centre line turbulence intensity data of Corrsin [8] for rescaling of the present data, and a centre line turbulence intensity of 0.025 for fully developed pipe flow data published by Fuchs [5] and Sheriff and O'Kane [2], the comparative data of Fig. 8 are obtained.

A trend of the three different sets of data towards $Pr_t = 0.9$ at large Reynolds number is apparent. It is clear, however, that Pr_t is not a unique function of Reynolds number as suggested by equation (1). This could be due to the lack of use of a local length scale in the definition of turbulent Reynolds number. However, quite large changes in such length scales from those used here would be required in order to reduce the results to a unique functional relationship between Pr_t and $Re_{u'}$.

Good consistency of results now available is noted when considering Pr_t as a function of ε_{H0}/α rather than U_0 or Reynolds number directly (Fig. 9). The three different sets of results can be represented well by equation (17). While this function retains the dependence of Pr_t on velocity through the velocity dependence of ε_{H0}, it does exclude the case of $\varepsilon_{H0}/\alpha = 0$ at finite Reynolds numbers reported by Sheriff and O'Kane [2]. As pointed out by these authors, omission of this limit is not of practical significance, since turbulent heat transfer becomes neg-

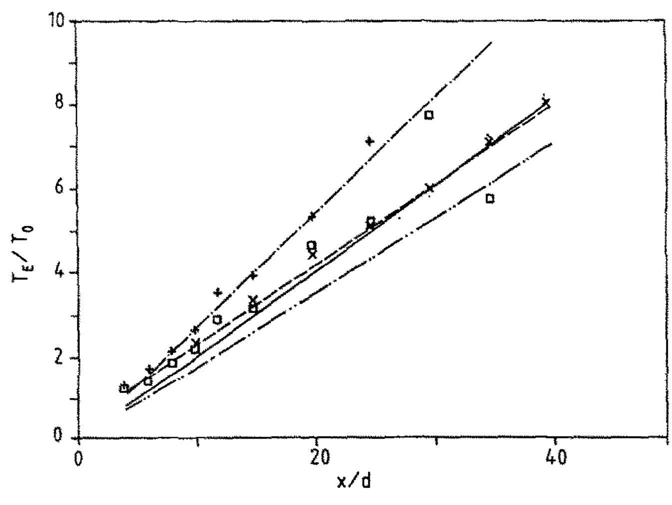

FIG. 4. Centre line temperature decay. (+) $U_E = 0.75$ m s^{-1}; (□) $U_E = 1.25$ m s^{-1}; (×) Corrsin [8]. (-·-) Equation (15), $A_3 = 3.7$, $x_{03}/d = 0$. (--) Equation (15), $A_3 = 5.3$, $x_{03}/d = 2.1$. (——) Chen and Rodi [11]. (-··-) Wilson and Danckwerts [9].

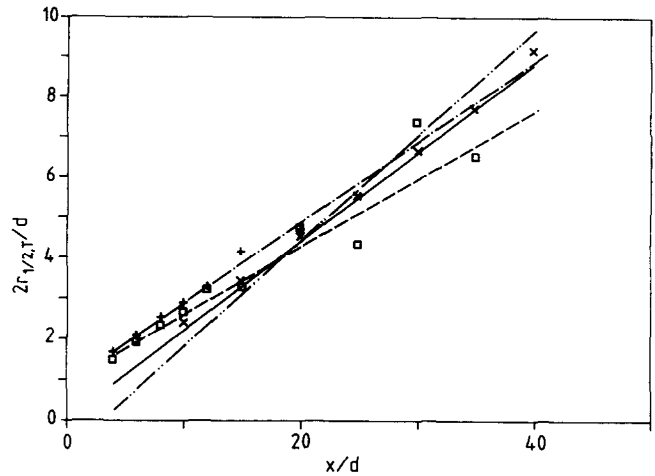

FIG. 5. Half value radii of temperature profiles. (+) $U_E = 0.75$ m s^{-1}; (□) $U_E = 1.25$ m s^{-1}; (×) Corrsin [8]. (-·-) Equation (14), $A_4 = 0.20$, $x_{04}/d = 4.4$. (--) Equation (14), $A_4 = 0.17$, $x_{04}/d = 5.6$. (——) Chen and Rodi [11]. (-··-) Wilson and Danckwerts [9].

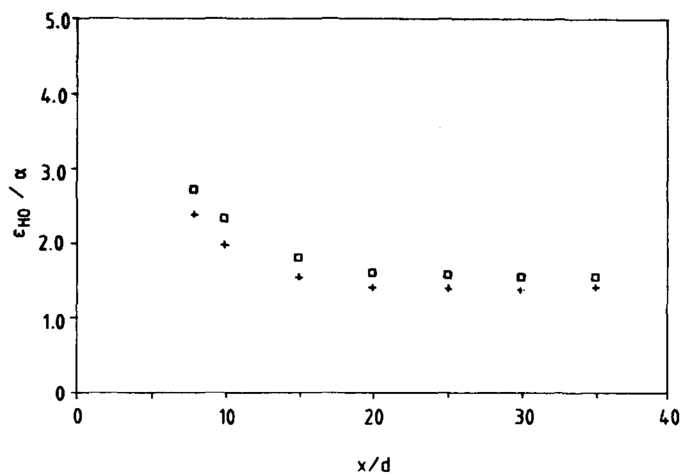

FIG. 6. Eddy diffusivities of heat along the jet centre line. (+) $U_E = 0.75$ m s^{-1}; (□) $U_E = 1.25$ m s^{-1}.

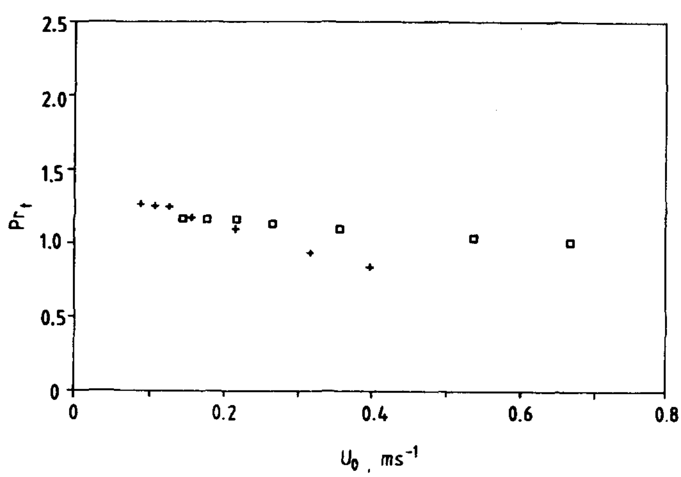

FIG. 7. Turbulent Prandtl numbers as a function of centre line velocity. (+) $U_E = 0.75$ m s^{-1}; (□) $U_E = 1.25$ m s^{-1}.

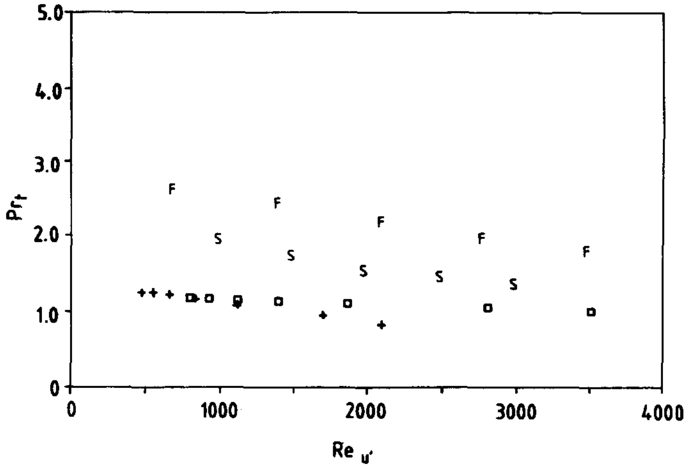

FIG. 8. Turbulent Prandtl numbers for various liquid sodium flows as a function of turbulent Reynolds number. (+) $U_E = 0.75$ m s^{-1}; (□) $U_E = 1.25$ m s^{-1}. F, Fuchs [5]; S, Sheriff and O'Kane [2].

ligible for very low values of ε_{H0}/α and can, therefore, be neglected:

$$Pr_t = 1.8\exp(-1.5\varepsilon_{H0}/\alpha)+0.9. \qquad (17)$$

The utility of this expression lies in the fact that once the eddy diffusivity of momentum has been found from turbulence flow models such as the k–ε one, equation (17) becomes an expression in ε_{H0}/α. Solution of this equation for ε_{H0}/α is readily possible by standard numerical analysis techniques such as Newton's method. The resultant value of ε_{H0}/α can then be used in the thermal energy equation to solve for the temperature field.

CONCLUDING REMARKS

Combination of the above jet flow results with those previously available indicates that at low flow velocities, turbulent Prandtl number is velocity dependent and increases with decreasing velocity. The simple inverse relationship proposed by Jischa [4] and given by equation (1) could not be verified with the jet flow measurements reported here, perhaps due to the difficulty of direct measurement of a meaningful length scale for a mean or turbulence velocity based Reynolds number in liquid sodium and the inability to reach sufficiently low velocities, but an alternative relationship for Pr_t, equation (17), has been found to give a good fit to available experimental data obtained above and that obtained by others in pipe flow. This expression contains the velocity or Reynolds number dependence through the eddy diffusivity, which contains considerably more flow information than mean or turbulent velocity based Reynolds numbers.

Combination of results for the three different liquid sodium flows shows that an increase in turbulent Prandtl number becomes noticeable when $\varepsilon_H/\alpha < 2.5$, which can also be expressed as $\varepsilon_M/\nu < 2.5Pr_t/Pr$ or simply $\varepsilon_m/\nu < 2/Pr$, since $Pr_t \geqslant 0.9$ in equation (17) and the exact point of deviation of Pr_t from 0.9 is somewhat arbitrary. Very large turbulent Prandtl numbers are due to high fluid conductivity leading to

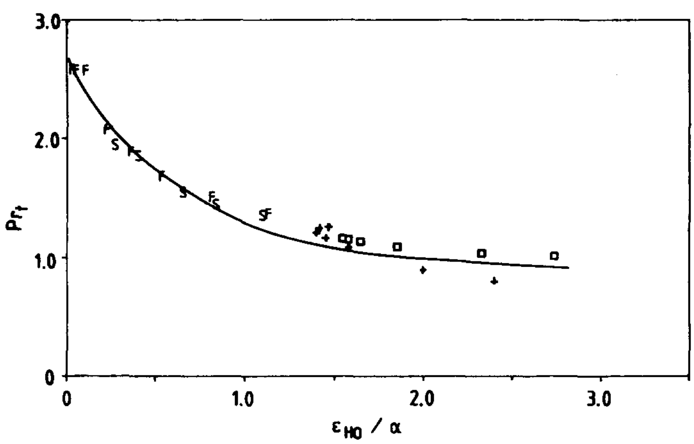

FIG. 9. Turbulent Prandtl number for various liquid sodium flows as a function of eddy diffusivity of heat. (+) $U_E = 0.75$ m s^{-1}; (□) $U_E = 1.25$ m s^{-1}. F, Fuchs [5]; S, Sheriff and O'Kane [2].

an inertial-conductive flow regime where the momentum field is still turbulent but no longer produces significant turbulent heat transport even though a temperature gradient still exists. Physically, this will occur where heat transfer is by conduction, that is, molecular diffusion. Such an inertial-conductive regime is clearly incompatible with the turbulent diffusion assumption inherent in the turbulent Prandtl number.

These considerations lead to the natural conclusion that simple turbulent Prandtl number modelling for low Prandtl number fluids may not lead to reliable predictive schemes for very low velocities or flows with low eddy diffusivities. It may be thought that the use of transport equations for δ' and ε_H as proposed by Nagano and Kim [12] will lead to a predictive scheme free of a turbulent Prandtl number. The prospects are not good, however, since the constant of proportionality used to relate eddy diffusivity of heat to turbulent kinetic energy, temperature fluctuation level and their respective dissipation rates contains the turbulent Prandtl number and the thermal-to-velocity time scale ratio. Available results indicate that Pr_t is a function of velocity and recent results by Bremhorst *et al.* [13] have shown that the time scale ratio is a function of molecular Prandtl number. This constant of proportionality should, therefore, be a functional relationship.

Acknowledgements—The initial support of one author (KB) by the Alexander von Humboldt Foundation and the efforts of Professor U. Müller (KfK/IRB) are gratefully acknowledged.

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NOMBRES DE PRANDTL TURBULENTS DETERMINES EXPERIMENTALEMENT DANS LE SODIUM LIQUIDE A FAIBLES NOMBRES DE REYNOLDS

Résumé—Des nombres de Prandtl turbulents sont déduits des mesures de distribution de vitesse moyenne et de température moyenne et de leur rayon de demi-valeur pour un jet de sodium liquide dans un fluide au repos. Les résultats obtenus indiquent une dépendance vis-à-vis de la vitesse et sont semblables à ceux rapportés dans la littérature pour les écoulements en conduite. La dépendance à la vitesse, qui est supposée être due à la transition vers un régime d'écoulement inertiel-conductif où la diffusion turbulente thermique inhérente au nombre de Prandtl turbulent devient prononcée à faible vitesse. Une combinaison des données du nouvel écoulement de jet et de l'écoulement existant de sodium liquide en conduite fournit une relation fonctionnelle simple entre le nombre de Prandtl turbulent et la diffusivité thermique turbulente qui peut être obtenue pour ε_H si ε_M est connu. Les résultats combinés montrent que, pour le sodium liquide, des accroissements sensibles du nombre de Prandtl turbulent apparaissent quand $\varepsilon_H/\alpha < 2,5$ c'est-à-dire quand $\varepsilon_M/\nu < 2/Pr$.

EXPERIMENTELL ERMITTELTE TURBULENTE PRANDTL-ZAHLEN IN FLÜSSIGEM NATRIUM BEI KLEINEN REYNOLDS-ZAHLEN

Zusammenfassung—An einem Strahl aus flüssigem Natrium, der in ein praktisch ruhendes Fluid ausströmt, wurden Messungen ausgeführt. Aus den ermittelten Verteilungen der mittleren Strömungsgeschwindigkeit und der mittleren Temperatur sowie aus deren Halbwert-Radius werden turbulente Prandtl-Zahlen abgeleitet. Die Ergebnisse zeigen einen Einfluß der Strömungsgeschwindigkeit und sind im übrigen ähnlich wie diejenigen aus der Literatur für Rohrströmungen. Die Geschwindigkeitsabhängigkeit ist bei kleinen Geschwindigkeiten besonders ausgeprägt. Von ihr wird angenommen, daß sie durch Übergang in das Gebiet einer trägheits/leitungsbestimmten Strömung zustande kommt, wo die turbulente Diffusion von Wärme als Teil der turbulenten Prandtl-Zahl nicht mehr existiert. Aus der Kombination der neuen Daten für die Strahlströmung und vorhandenen Daten für die Rohrströmung von flüssigem Natrium ergibt sich eine einfache Beziehung zwischen der turbulenten Prandtl-Zahl und der turbulenten Temperaturleitfähigkeit, aus der leicht ε_H berechnet werden kann wenn ε_M bekannt ist. Die kombinierten Ergebnisse zeigen, daß die turbulente Prandtl-Zahl für flüssiges Natrium bei Strömungsbedingungen $\varepsilon_H/\alpha < 2{,}5$ signifikant zunimmt, d.h. wenn $\varepsilon_M/\nu < 2/Pr$ ist.

ЭКСПЕРИМЕНТАЛЬНОЕ ОПРЕДЕЛЕНИЕ ТУРБУЛЕНТНЫХ ЧИСЕЛ ПРАНДТЛЯ В ЖИДКОМ НАТРИИ ПРИ НИЗКИХ ЧИСЛАХ РЕЙНОЛЬДСА

Аннотация—На основе измерений распределений средней скорости и температуры и их значений на половине радиуса струи жидкого натрия, истекающей в неподвижную среду, определены значения турбулентного числа Прандтля. Полученные результаты показывают, что они зависят от скорости и аналогичны имеющимся в литературе для течений в трубах. Зависимость от скорости, обусловленная переходом к инерционнокондуктивному режиму течения, когда отсутствует характерная для турбулентного числа Прандтля турбулентная диффузия тепла, становится более существенной при низких скоростях. Новые результаты для струйного течения вместе с имеющимися данными для течений в трубах в случае жидкого натрия позволяют установить простую функциональную зависимость между турбулентным числом Прандтля и турбулентной температуропроводностью, которая легко решается при известных значениях ε_H и ε_M. Полученные результаты боказывают, что в случае с жидким натрием происходит существенное увеличение турбулентного числа Прандтля при условии, что $\varepsilon_H/\alpha < 2{,}5$, т.е., когда $\varepsilon_M/\nu < 2/Pr$.